Phi 201: Precept 10

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Exercise 1. Translate the following. For the first three say why they're false (i.e. say which condition of the sentence fails):

(a) The book in firestone is a logic text.

(b) The professor of Phi201 is from California.

(c) The object causing perturbations in Mercury's orbit is a planet.

(d) Alejandro has exactly three cats.

(e) Kyle is the head preceptor.

(f) The greatest prime is even.

Exercise 2. Russell claims there is a scope ambiguity in some sentences. There are two readings of each of the following. Translate both readings, say why they're different, and note the historical significance of the first three.

(a) The present king of France is not bald.

(b) The planet causing perturbations in Mercury's orbit doesn't exist.

(c) Snow is wondering if the morning star is the evening star.

(c) Sten believes the person sitting over there is famous.

Models. To show the consistency of a set of sentences, we present models. We say that a set of sentences is *satisfiable* just in case there is a possible situation in which the sentences are all true (i.e. Δ is *satisfiable* (or has a model) iff there is a model M such that for all $\delta \in \Delta$: $M \vDash \delta$). A *model* in our sense consists of a non empty set called the *domain*, which specifies the particular objects that exist[1] and an interpretation of function and predicate symbols. For our purposes now, we care about the interpretation of predicate symbols. That is, *extensions of predicates*, which specify which objects have which properties. So for example, if we want to know if the sentence $\exists x P(x) \wedge \exists x F(x)$ is consistent, we specify a domain, which is a set of individuals $D = \{a, b\}$ and the extension of predicates $Ext(P) = \{a\}$ and $Ext(F) = \{b\}$. These are simply the predicates that we care about in this situation.[2] What we've just said is that there is a possible situation in which a and b are the only

[1] For our purposes, the domain is never empty.

[2] If they are one-placed predicates, then the extensions are subsets of the domain (e.g. $Ext(P) = \{a\} \subseteq \{a, b\} = D$.) If they are two-placed, they will be subsets of $D \times D$, and so on.

objects and a is the only P and b is the only F. This means that something is P and something is F. Comprehension question: Does this model ensure that a and b are distinct objects? Is there a simpler model and if so provide it?

Exercise 3. Note that $\forall$ distributes over $\wedge$ and $\exists$ distributes over $\vee$. Give examples that satisfy each of the following:

1. $\forall x(Fx \wedge Gx) \dashv\vdash \forall xFx \wedge \forall xGx$
2. $\exists y(Fy \vee Gy) \dashv\vdash \exists yFy \vee \exists yGy$

Countermodels: To prove that a sentence does *not* logically follow from a set of sentences, we give a countermodel, which is a model in which the premises are true and the conclusion is false. For example, say we want to show that $\exists xFx \nvdash \forall xFx$. We can prove this by giving a countermodel as follows: Let $D = \{a, b\}$ and $Ext(F) = \{a\}$. This means that there are two objects a and b and a is the only F. Since b is not an F, not all things are F. That is, *there is no proof* from $\exists xFx$ to $\forall xFx$. Comprehension question: Are the sentences satisfiable? If so, how? How do we show that a sentence ϕ is consistent with/inconsistent with/provable from/not provable from a set of sentences Δ?

Exercise 4. Note the following distributions of $\forall$ over $\vee$:

$$\forall xFx \vee \forall xGx \vdash \forall x(Fx \vee Gx) \text{ but } \forall x(Fx \vee Gx) \nvdash \forall xFx \vee \forall xGx$$

Give an example that satisfies the sequent and a counterexample to the invalidity.

Exercise 5. Note the following distribution of $\exists$ over $\wedge$:

$$\exists y(Fy \wedge Gy) \vdash \exists yFy \wedge \exists yGy \text{ but } \exists yFy \wedge \exists yGy \nvdash \exists y(Fy \wedge Gy)$$

Give an example that satisfies the sequent and a counterexample to the invalidity.

Exercise 6. Provide a set-theoretic model showing that $\forall xP(x) \to \forall xQ(x)$ does not imply $\forall x(P(x) \to Q(x))$.

Exercise 7. Show that $\exists x(Px \to Qx) \nvdash \exists xPx \to \exists xQx$.

Exercise 8. Show that $\forall x\exists yRxy$ does not imply $\forall x\exists yRyx$.

Translations.

The F is G: $\exists x(Fx \wedge \forall y(Fy \to x = y) \wedge Gy)$

There are at least two F's: $\exists x\exists y(Fx \wedge Fy \wedge x \neq y)$

There are at most two F's: $\neg\exists x\exists y\exists z(Fx \wedge Fy \wedge Fz \wedge x \neq y \wedge x \neq z \wedge y \neq z)$

There are exactly two F's: $\exists x\exists y(Fx \wedge Fy \wedge \forall z(Fz \to (x = z \vee y = z)) \wedge x \neq y)$