Phi 201: Precept 3

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General rules for adding dependency numbers (numbers of the left-most column) are as follow:

(i) if the line of your proof is a premise (or supposition), copy the number of that line and *only* the number of that line as a dependency number (premises and suppositions *always* have only one number).

(ii) if the line of your proof is not a premise (or supposition), inherit the dependency numbers from your justification numbers (e.g. if you write down '3,4 MP' then you copy the dependency numbers from lines 3 and 4.), *unless* you are discharging (e.g. CP, ∨E,...).

With this method, as opposed to the one presented in the early lectures, every line of the proof is a valid deduction.

Exercise 1. Is the following a valid? Why or why not? What does the proof prove (if anything)?

1	(1)	$p \to q$	A
2	(2)	$(q \to r) \to (p \to r)$	A

Exercise 2. Is the following a valid? Why or why not? What does the proof prove (if anything)?

1	(1)	$p \to q$	A
2	(2)	$q \to r$	A
3	(3)	q	A
4	(4)	p	A
2, 3	(5)	r	2,3 MP
2, 3	(6)	$p \to r$	4,5 CP
3	(7)	$(q \to r) \to (p \to r)$	2,5 CP

Exercise 3. Give a proof for the following
$p \rightarrow q \vdash (q \rightarrow r) \rightarrow (p \rightarrow r)$

Exercise 4. Why is the following invalid? What does the proof prove (if anything)?

1	(1)	$p \wedge (q \vee r)$	A
1	(2)	p	$1 \wedge$ E
1	(3)	$q \vee r$	$1 \wedge$ E
4	(4)	q	A
5	(5)	r	A
4, 5	(6)	$q \wedge r$	$4,5 \wedge$ I
4, 5	(7)	q	$6 \wedge$ E
1, 4, 5	(8)	$p \wedge q$	$2,7 \wedge$ I
1, 4, 5	(9)	$(p \wedge q) \vee (p \wedge r)$	$8 \vee$ I
1	(10)	$(p \wedge q) \vee (p \wedge r)$	1,3,4,5,9$\vee$ E

Exercise 5. Give a proof for the following
$p \wedge (q \vee r) \vdash (p \wedge q) \vee (p \wedge r)$

Exercise 6. Why is the following invalid What does the proof prove (if anything)?

1	(1)	$(p \vee q)$	A
2	(2)	p	A
3	(3)	q	A
2, 3	(4)	$p \wedge q$	$2,3\wedge$ I
2, 3	(5)	p	$4 \wedge$ E
1	(6)	p	1,2,2,3,5 $\vee$E

Everyday proof by cases example. My boyfriend and I realized yesterday that our parking meter had just run out. I argued in the following way that there was no point in running back (logicians argue this way; don't date one): "At this point: either we've already gotten a ticket or we haven't. If we've gotten a ticket, we won't get another one in the time it takes us to get to the car, so rushing would serve no purpose. If we haven't gotten a ticket in the past several hours, it is extremely unlikely that we will get one in the next few minutes, so again, rushing would be pointless. In either case, there's no need to rush." He responded with the following counterargument (clearly I've had an impact): "Either we are going to get a ticket in the next few minutes or we aren't. If we are, then rushing might prevent it, which would be a good thing. If we aren't then it will still be good exercise and will also show our respect for the law, both of which are good things. So in either case, rushing back to the car is a good thing to do." Robbie won the argument.