Phi 201: Precept 8

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Exercise 1a. The following has three invalid inferences. Which steps are invalid and why?

1	(1)	$\neg\exists x\forall y(Fx \rightarrow Gy)$	A
2	(2)	$\neg\forall y(Fa \rightarrow Gy)$	A
2	(3)	$\neg(Fa \rightarrow Gb)$	2 UE
2	(4)	$Fa \wedge \neg Gb$	3 SI(S) MC
2	(5)	Fa	4 $\wedge$E
2	(6)	$\forall x Fx$	5 UI
1	(7)	$\forall x Fx$	1,2,6 EE

Exercise 1b. The following has one invalid inference. Locate which one, say why, and propose a fix.

1	(1)	$\exists y(Fa \rightarrow Gy)$	A
2	(2)	$Fa \rightarrow Gb$	A
3	(3)	$\neg\exists y Gy$	A
3	(4)	$\forall y \neg Gy$	3 SI(S) QDM
3	(5)	$\neg Gb$	4 UE
$2,3$	(6)	$\neg Fa$	2,5 MT
$2,3$	(7)	$Fa \rightarrow Gb$	6 SI(S) NP
$1,3$	(8)	$Fa \rightarrow Gb$	1,2,7 EE

Exercise 2. Note that $\forall$ distributes over $\wedge$ and $\exists$ distributes over $\vee$. Give examples that satisfy each of the following theorems:

1. $\forall x(Fx \wedge Gx) \dashv\vdash \forall x Fx \wedge \forall x Gx$
2. $\exists y(Fy \vee Gy) \dashv\vdash \exists y Fy \vee \exists y Gy$

Exercise 3. Note the following distributions of $\forall$ over $\vee$:

$$\forall xFx \vee \forall xGx \vdash \forall x(Fx \vee Gx) \text{ but } \forall x(Fx \vee Gx) \nvdash \forall xFx \vee \forall xGx$$

Give an example that satisfies the theorem and a counterexample to the invalidity.

Exercise 4. Note the following distribution of $\exists$ over $\wedge$:

$$\exists y(Fy \wedge Gy) \vdash \exists yFy \wedge \exists yGy \text{ but } \exists yFy \wedge \exists yGy \nvdash \exists y(Fy \wedge Gy)$$

Give an example that satisfies the theorem and a counterexample to the invalidity.

Set Theory
$x \in y$: x is an element of y
$x \subseteq y$: x is a subset of y. $x \subseteq y$ iff $\forall z(z \in x \to z \in y)$.

A few axioms of set theory

1. axiom of extensionality: $\forall x \forall y(\forall z(z \in x \leftrightarrow z \in y) \to x = y))$ use the law of identity to prove the converse
2. null set axiom: $\exists x \forall y(y \in x \leftrightarrow y \neq y)$

Informal Arguments
We define $A \cap B$ to be the set of elements that are in both A and B, and $A \cup B$ to be the set of elements that are in either A or B. That is:

- $\forall x(x \in A \cap B \leftrightarrow (x \in A \wedge x \in B))$
- $\forall x(x \in A \cup B \leftrightarrow (x \in A \vee x \in B))$

1. Prove that $A \cap B = B \cap A$ (i.e. $\cap$ is commutative)
2. Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$