Exercise 1: Let $a \in \emptyset \cup A$. Then either $a \in \emptyset$ or $a \in A$. Since nothing is an element of $\emptyset$ by axiom $a \notin \emptyset$ and so $a \in A$. Hence, $\emptyset \cup A \subseteq A$

For the other direction, let $a \in A$ then by disj introduction $a \in A$ or $a \in \emptyset$. or commutes so $a \in \emptyset \cap A$ and hence $A \subseteq \emptyset \cup A$ and we are done.

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Exercise 2: The exercise is trivial.
$a \in A$ so, $a \in A$ _ done.

$\blacksquare$

Exercise 3: Similarly, trivial.

$\blacksquare$

Exercise 4: Let $P(n) :=$

$$0 + 1 + \ldots + n = \frac{n(n+1)}{2}$$

Base Case: $P(0)$

$$0 = \frac{0(0+1)}{2} = 0 \checkmark$$

Inductive Case: Assume $P(k)$ <to show $P(k+1)$>

$$0 + 1 + \ldots + k = \frac{k(k+1)}{2} \quad \text{by ind hypoth.}$$

$$\Rightarrow 0 + 1 + \ldots + k + (k+1) = \frac{k(k+1)}{2} + (k+1)$$

$$\Rightarrow 0 + 1 + \ldots + (k+1) = \frac{(k+1)(k+2)}{2}$$, which is $P(k+1)$

Therefore by induction, $0 + 1 + \ldots + n = \frac{n(n+1)}{2}$ for all $n \in \mathbb{N}$.

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Exercise 5: ON HANDOUT.